I Semester B.C.A. Degree Examination, Nov./Dec. 2017 (2014-15 and Onwards) (F + R) (CBCS) BCA - 105 T: DISCRETE MATHEMATICS

Time: 3 Hours

Max. Marks: 100

Instruction: Answerall Sections.

SECTION - A

I. Answer any ten of the following:

 $(10 \times 2 = 20)$

- 1) If $A = \{2, 3, 4, 5\}$ and $B = \{0, 1, 2, 3\}$, find $A \cap B$.
- 2) If $A = \{x^2 5x + 6 = 0, x \in N\}$ and $B = \{3, 4, 5\}$, find $A \times B$.
- 3) Define contradiction.
- 4) Define unit matrix with example.

5) If
$$A = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$
 and $B = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$, find $2A + 3B$.

- 6) Find the characteristic roots of the matrix $A = \begin{bmatrix} 3 & 0 \\ 2 & 5 \end{bmatrix}$.
- 7) Prove that $\log_{3a} 2a \cdot \log_{4a^2} 3a = \frac{1}{2}$.
- 8) If ${}^{n}C_{30} = {}^{n}C_{5}$, find 'n'.
- 9) Define group.
- 10) If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} 2\hat{j} + \hat{k}$, find $|2\vec{a} + \vec{b}|$.
- 11) Find the distance between the points A(2, -3) and B(4, 5).
- 12) Write the slope of the line 4x 3y + 2 = 0.



SECTION - B

II. Answer any six of the following:

(6×5=30)

- 13) In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?
- 14) If $f: R \to R$ is defined by f(x) = 4x + 5 prove that f is one-one and onto.
- 15) Prove that $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.
- 16) Prove that $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$.
- 17) Write the converse, inverse and contrapositive of "If two triangles are congruent, then they are similar".
- 18) If $A = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$ B = [2 3 5] prove that (AB)' = B'A'.
- 19) If $A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then find A^{-1} using Cayley-Hamilton theorem.
- 20) Solve 5x + 2y = 4, 7x + 3y = 5 using Cramer's rule.

SECTION - C

III. Answer any six of the following:

 $(6 \times 5 = 30)$

- 21) If $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$, then prove that $a^2 + b^2 = 7ab$.
- 22) Prove that the set $G = \{1, -1, i, -i\}$ is a group under multiplication.
- 23) Prove that H = {0, 2, 4} is a subgroup of G = {0, 1, 2, 3, 4, 5} under addition Modulo 6.
- 24) How many different words can be formed with the letters of the word "MISSISSIPPI"? In how many of these four I's do not come together?
- 25) If ${}^{2n}C_3 : {}^{n}C_2 = 44 : 3$ find n.
- 26) If $\vec{a} = 2\hat{i} + \hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} \hat{j} + 2\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 4\hat{k}$ find $\vec{a} \cdot (\vec{b} \times \vec{c})$.
- 27) Find the area of the triangle whose vertices are A(3, 2, 1) B(4, -1, 2) and C(-1, 3, 2) using vector method.
- 28) Find the value of m if $\vec{a} = m\hat{i} 3\hat{j} + 4\hat{k}$ $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + \hat{k}$ are coplanar.



SECTION - D

IV. Answer any four of the following:

 $(4 \times 5 = 20)$

- 29) Prove that the points A(3, -4), B(4, 2), C(5, -4) and D(4, -10) form vertices of a rhombus.
- 30) If a vertex of triangle is (1, 1) and the mid-point points of two sides through this vertex are (-1, 2) and (3, 2) then find the centroid of the triangle.
- 31) Find the acute angle between the lines 2x y + 13 = 0 and 2x 6y + 7 = 0.
- 32) The angle between two lines is $\frac{\pi}{4}$ and the slope of one line is $\frac{1}{2}$. Find the slope of the other line.
- 33) Find the point of intersection of the straight lines 3x 4y 1 = 0 and 5x 7y 1 = 0.
- 34) Prove that the point (-1, 3) is equidistant from the lines x + y 3 = 0 and 7x y + 5 = 0.